

Mapping the Shifting Dynamics of Mathematics Learning in the Artificial Intelligence Era: A Psychometric Network Analysis of the Community of Inquiry

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Abstract

The integration of generative artificial intelligence (AI) into undergraduate mathematics classrooms has begun to shift the dynamics of the learning ecosystem — reconfiguring how reflective interaction, metacognitive regulation, and cognitive engagement relate to one another within the Community of Inquiry (CoI) framework. Latent-variable structural models have documented that this reconfiguration exists, but they cannot pinpoint which specific learner behaviors carry the shift. This study applies psychometric network analysis to produce a behavior-level map of the reconfigured ecosystem. Using cross-sectional data from 200 undergraduate mathematics education students at Universitas Negeri Makassar who used conversational AI during a differential calculus course, we estimated a regularized partial-correlation network over 19 items from an AI-Integrated CoI survey using the Graphical Lasso with cross-validated penalty selection. Node strength and bridge strength were computed to identify hub and boundary-spanning behaviors, and bootstrap resampling (300 iterations) was used to assess stability. The network retained 87 edges (density = 0.51). The four items with the highest bridge strength were RMP2 (using AI to check one's own solution steps — 0.45), LP3 (monitoring understanding of AI



explanations — 0.45), RMP5 (asking follow-up questions to deepen understanding — 0.40), and LP6 (reflecting on whether AI is helping — 0.39). Three of these four top-ranked items describe monitoring and verification behaviors that occur during the AI interaction rather than the post-hoc evaluation of AI outputs most often emphasized in AI-literacy discourse. Bootstrap Spearman correlation between bootstrapped and reference bridge rankings averaged 0.80, indicating moderate stability. The findings reframe the AI-induced shift as a shift toward in-interaction epistemic monitoring rather than a shift toward output-checking vigilance.

Keywords: Shifting learning dynamics; Generative artificial intelligence; Community of inquiry; Network psychometrics; Metacognitive monitoring; Calculus learning.

INTRODUCTION

The rapid arrival of generative artificial intelligence (AI) in undergraduate mathematics classrooms has begun to shift the dynamics of the learning ecosystem. Where the explanations, procedural verifications, and adaptive hints traditionally supplied by instructors and peers now stream instantly from a conversational agent, the balance among the classical presences of the Community of Inquiry (CoI) framework — teaching, cognitive, and social — is reconfigured (Garrison et al., 2000; Kasneci et al., 2023; Lodge et al., 2023). Recent CoI research has begun documenting this reconfiguration by introducing a reflective machine presence dimension that captures interaction patterns unique to AI use (Bearman et al., 2022), and by testing whether the effect of this new presence on cognitive engagement is mediated by learner metacognition (Zawacki-Richter et al., 2019). These structural findings tell us that a shift is occurring, and they estimate its aggregate magnitude.

The latent-variable framing that has yielded these findings, however, cannot yet tell us where in the learner's actual behavioral repertoire the shift is happening. By treating the items on a scale as fungible indicators of an underlying construct, structural equation modeling (SEM) aggregates away information about which particular behaviors — asking the AI for a step-by-step verification, planning a strategy before typing a prompt, or evaluating whether an explanation actually helped — are most consequential in binding the reconfigured system together. For an instructor asking a concrete pedagogical question, a coefficient for a latent variable is only partly useful. What is needed is a behavior-level map: which specific actions serve as the load-bearing bridges of the AI-mediated learning ecosystem.



Psychometric network analysis provides such a map (Epskamp & Fried, 2018). Instead of estimating latent variables, it models each survey item as a node and each edge as a partial correlation between two items after controlling for all others. In this graph, certain behaviors emerge as more central than others: they have many strong partial correlations with many neighbors, and they may serve as bridges between clusters of items belonging to different constructs (Jones et al., 2019). Identifying such bridge behaviors matters for the shifting-dynamics question because the shift is not distributed evenly across the ecosystem: a small set of behaviors may hold the reconfigured system together, and interventions targeting these behaviors would propagate their effect across the whole learning environment.

The present study applies psychometric network analysis to a 19-item AI-Integrated Community of Inquiry (AI-Col) survey administered to undergraduate calculus students. Four subscales are represented: Reflective Machine Presence (RMP), Cognitive Delegation (COG_DEL), Learner Presence (LP), and Cognitive Presence (CP). Two questions guide the analysis. First, which specific behaviors are the most central hubs of the reconfigured AI-Col network (node-strength centrality)? Second, which behaviors function as boundary-spanning bridges across the four subscales (bridge-strength centrality)? Because small-sample networks can produce unstable estimates (Epskamp et al., 2018), we also conducted a bootstrap analysis to test the stability of the centrality rankings and to communicate honestly which findings are and are not robust.

METHOD

Participants and instrument

Two hundred ($n = 200$) undergraduate students from the Mathematics Education Study Program at Universitas Negeri Makassar completed the AI-Col survey after a differential calculus problem-solving activity that required interacting with a conversational AI (ChatGPT or Gemini). The AI-Col is a 19-item Likert instrument (1 = strongly disagree, 5 = strongly agree) organized into four subscales: RMP (5 items on reflective AI use), COG_DEL (2 items on delegating cognitive work to the AI), LP (6 items on metacognitive regulation while using AI), and CP (6 items on cognitive engagement in mathematics). Internal consistency was acceptable for all subscales: RMP $\alpha = 0.889$, COG_DEL $\alpha = 0.745$, LP $\alpha = 0.850$, CP $\alpha = 0.853$.

Network estimation

Because the number of parameters in a full covariance matrix on 19 items (190) is large relative to $n = 200$, the maximum-likelihood partial correlation matrix is prone to overfitting. We



therefore estimated the network with the Graphical Lasso (Friedman et al., 2008), which imposes an L1 penalty on the precision matrix, shrinking small entries exactly to zero and thereby producing a sparse and stable graph. The tuning penalty was selected automatically by 5-fold cross-validation (GraphicalLassoCV in scikit-learn). Partial correlations were computed from the resulting precision matrix as $\pi_{ij|rest} = -\Omega_{ij} / \sqrt{(\Omega_{ii} \cdot \Omega_{jj})}$, where Ω denotes the estimated precision matrix.

Centrality metrics

Two centrality metrics were computed for each of the 19 items. Node strength is the sum of absolute edge weights connecting the item to all its neighbors, indicating overall interconnectedness in the network. Bridge strength (Jones et al., 2019) is the sum of absolute edge weights connecting the item to nodes belonging to a different subscale; a high value marks the item as a translator between subscales, and interventions targeting such items are theoretically positioned to propagate their effect across the whole system.

Stability analysis

To evaluate the robustness of the centrality rankings, we performed non-parametric bootstrap resampling with 300 iterations (Epskamp et al., 2018). For each bootstrap sample, the graphical lasso was re-estimated with 3-fold internal cross-validation, and bridge strengths were recomputed. We report (a) the mean and standard deviation of bootstrap bridge strengths for each item, (b) the proportion of bootstraps in which each item appeared among the top four bridges of that sample, and (c) the Spearman rank correlation between bootstrap and reference rankings.

RESULTS AND DISCUSSION

Network topology

The estimated network retained 87 edges out of 171 possible pairs (density = 0.51). Figure 1 shows the network topology. The four subscales form clearly separated local clusters — RMP items in the upper region, LP items across the middle, CP items in the lower region, and the two COG_DEL items (RMP6, RMP7) as an isolated pair in the upper left corner. This isolation of the delegation items visually confirms the near-zero zero-order correlation between reflective use and delegation reported in the companion structural analysis: cognitive delegation is a distinct behavioral cluster rather than a reversed form of reflective engagement.



Figure 1. Regularized Partial-Correlation Network of the 19 AI-Col Items
(Graphical Lasso with CV-selected penalty; node size $\propto$ bridge strength)

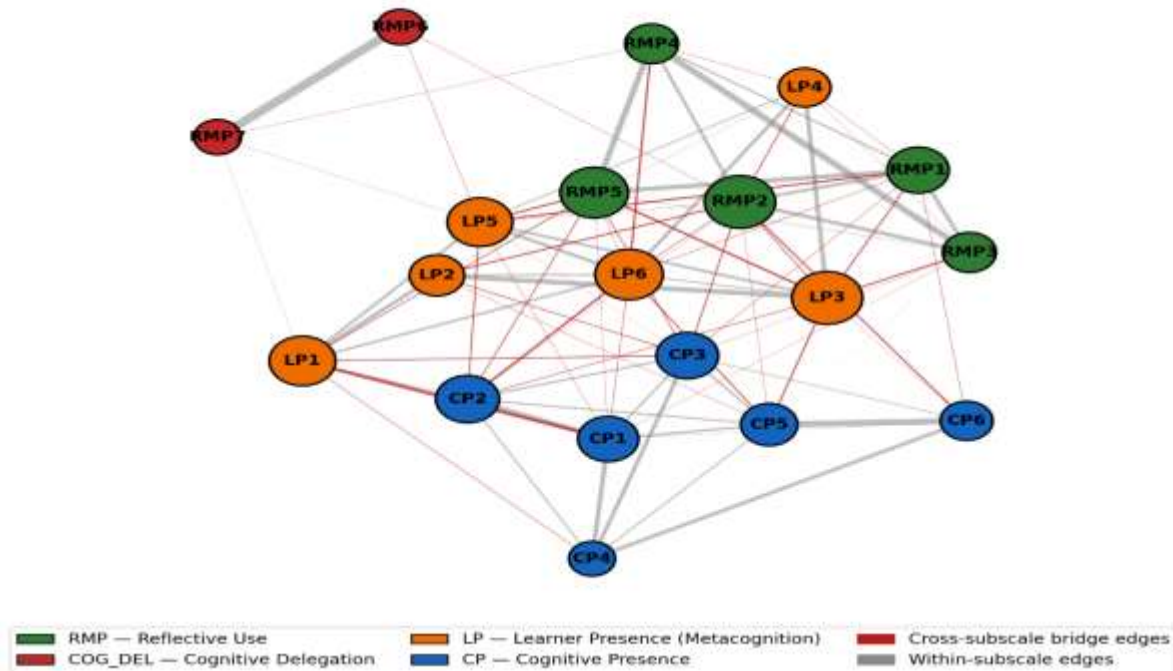


Figure 1. Regularized partial-correlation network of the 19 AI-Col items (Graphical Lasso).

Node-strength centrality: The hubs of the network

Node strength identifies which items are most densely connected to their neighbors, regardless of subscale. The top eight items by node strength are reported in Table 1.

Table 1. Top eight items by node-strength centrality (Graphical Lasso network)

Item	Subscale	Node strength	Behavior described
LP3	LP	1.12	Monitoring whether one understands the AI's explanations
LP6	LP	1.03	Reflecting on whether AI use is helping understanding
RMP2	RMP	0.96	Using AI to check one's own solution steps
RMP1	RMP	0.96	Asking the AI for the underlying logic, not the answer
RMP5	RMP	0.95	Asking follow-up questions to deepen understanding
CP1	CP	0.94	Exploring various ways of solving problems
RMP4	RMP	0.94	Asking the AI for alternative explanations or analogies
CP5	CP	0.92	Solving problems independently after using AI

The two most connected items in the entire network are LP items — LP3 and LP6 — both of which describe metacognitive-monitoring behaviors: monitoring comprehension during AI interaction (LP3) and reflecting afterwards on whether the interaction helped (LP6). RMP items concerned with verification and elaboration (RMP2, RMP1, RMP5, RMP4) form the immediately next tier. No COG_DEL item appears among the top eight; the delegation items are strongly connected to each other but sparsely connected to the rest of the network.

Bridge-strength centrality: The behaviors that connect the subscales

Table 2 reports the top ten items by bridge strength — the sum of edge weights connecting each item to nodes in a different subscale. Figure 2 visualizes the bridge strength for all 19 items.

Table 2. Top ten items by bridge-strength centrality (Graphical Lasso network)

Rank	Item	Subscale	Bridge strength	Behavior described
1	RMP2	RMP	0.45	Using AI to check one's own solution steps
2	LP3	LP	0.45	Monitoring whether one understands AI explanations
3	RMP5	RMP	0.40	Asking follow-up questions to deepen understanding
4	LP6	LP	0.39	Reflecting on whether AI use is helping understanding
5	LP1	LP	0.36	Planning a strategy before solving
6	LP5	LP	0.34	Evaluating AI responses before accepting them
7	CP2	CP	0.31	Connecting new concepts to previously learned concepts
8	CP3	CP	0.28	Understanding the reasoning behind each step
9	RMP1	RMP	0.28	Asking the AI for the underlying logic
10	CP1	CP	0.26	Exploring various ways of solving problems



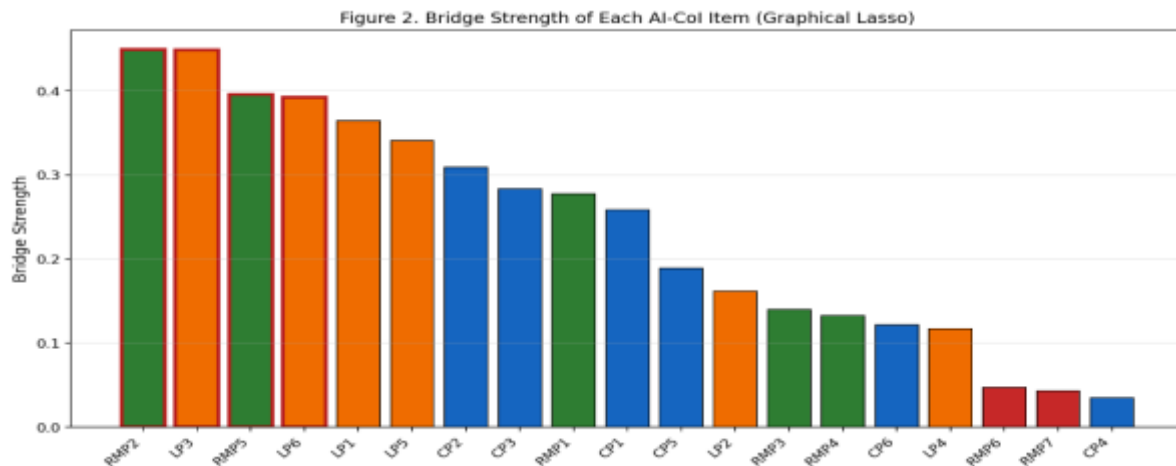


Figure 2. Bridge strength for each AI-Col item, colored by subscale.

Three of the top four bridge items describe monitoring and verification behaviors that occur during the AI interaction itself. RMP2 concerns using the AI to check one's own solution steps — a verification act performed during problem solving. LP3 concerns monitoring comprehension of the AI's explanations as they arrive — a monitoring act performed during interaction. RMP5 concerns asking follow-up questions to deepen understanding — an elaboration act performed during interaction. Only the fourth top bridge, LP6, describes a post-interaction behavior (retrospective reflection on whether the AI helped). LP5 — evaluating AI responses before accepting them, the item most often emphasized in the AI-literacy discourse as the definitional metacognitive behavior — appears at rank 6, below several less-emphasized behaviors.

This ordering suggests a reprioritization. The behaviors that most effectively connect reflective AI use, metacognitive regulation, and cognitive engagement are not primarily concerned with checking the AI's output for errors after the fact. They are concerned with in-interaction epistemic monitoring: using the AI to check one's own work, monitoring whether one is following the AI's reasoning, and asking follow-up questions when understanding is incomplete. Evaluation of AI output (LP5) and up-front planning (LP1) remain important as adjunct behaviors, but they are not the structural spine of the network.

Stability of the centrality rankings

Bootstrap analysis (300 iterations) showed moderate stability of the bridge rankings. The Spearman rank correlation between bootstrap and reference bridge rankings averaged 0.80 (SD = 0.17), with 88% of bootstrap samples exceeding $r = 0.70$. Item-level stability, reported as the

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proportion of bootstrap samples in which each item appeared among the top four bridges of that sample, was highest for LP3 (73%), RMP2 (66%), LP1 (52%), RMP5 (50%), and LP6 (49%). CP items dropped out of the top four in most bootstrap samples (CP2 = 22%, CP3 = 14%). These stability figures indicate that the identification of the top-four bridge set is reasonably robust — every item in our reported top four appears in at least half of the bootstrap samples — but that the exact ranking within the top four should be interpreted cautiously.

Cognitive delegation as a distinct cluster

A secondary but important observation concerns the position of the cognitive delegation items (RMP6, RMP7). Both items retain a strong within-pair connection but connect only weakly to the rest of the network (bridge strengths 0.049 and 0.044, respectively — near the bottom of Figure 2). This visually confirms what the near-zero zero-order correlation between reflection and delegation already suggested at the composite level: cognitive delegation is not the negative pole of reflective use but a distinct behavioral cluster that operates largely outside the main body of the AI-Col network.

Implications for instrument design and pedagogy

Two implications follow. For instrument design, the peripheral position of RMP6 and RMP7 reinforces recent calls to model delegation as its own construct rather than as the negative pole of reflection; reverse-scoring these items would collapse two independent dimensions into one and destroy informational value. For pedagogy, the concentration of bridge strength in RMP2, LP3, RMP5, and LP6 — three of them monitoring behaviors during AI interaction rather than after it — identifies four concrete teachable habits. Instead of framing AI literacy primarily as "do not accept the AI's answer uncritically" — a rule which the network suggests is not the load-bearing bridge — instructors can frame it as a routine of in-interaction monitoring: use the AI to check your own work as you go, monitor whether you are following the explanation, ask follow-up questions when you are not, and reflect afterwards on whether the arrangement helped. Each of these four habits corresponds to a specific item in the AI-Col instrument, making their acquisition both teachable and measurable.

CONCLUSION

A psychometric network analysis of 19 AI-Col items in a sample of 200 undergraduate calculus students, estimated with the Graphical Lasso and validated with bootstrap resampling,



identifies four behaviors — using the AI to check one's own steps (RMP2), monitoring comprehension of AI explanations (LP3), asking follow-up questions (RMP5), and reflecting on whether the interaction helped (LP6) — as the primary bridges connecting the subscales of the AI-integrated Community of Inquiry model. Three of these four are monitoring behaviors performed during the AI interaction rather than post-hoc evaluation of AI outputs. The results reframe the AI-induced shift in mathematics learning dynamics as a shift toward in-interaction epistemic monitoring rather than a shift toward output-checking vigilance. Limitations include the cross-sectional single-course design, reliance on self-report, and moderate rather than high bootstrap stability of the centrality rankings. Longitudinal and behavioral-trace validation of the identified bridge behaviors are important next steps.

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